

Measurement of Bose-Einstein Correlations in $e^+e^- \rightarrow W^+W^-$ Events at LEP

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Abstract

Bose-Einstein correlations in W-pair production at LEP are investigated in a data sample of 629 pb^{-1} collected by the L3 detector at $\sqrt{s} = 189\text{--}209 \text{ GeV}$. Bose-Einstein correlations between pions within a W decay are observed and found to be in good agreement with those in light-quark Z decay. No evidence is found for Bose-Einstein correlations between hadrons coming from different W's in the same event.

Submitted to *Phys. Lett. B*

Introduction

In hadronic Z decay, Bose-Einstein correlations (BEC) are observed as an enhanced production of identical bosons at small four-momentum difference [1, 2]. BEC are also expected within hadronic W decay (intra-W BEC). At LEP energies, in fully-hadronic W^+W^- events ($q\bar{q}q\bar{q}$) the W decay products overlap in space-time. Therefore, it is also natural to expect [3, 4] BEC between identical bosons originating from different W's (inter-W BEC). A comparison of BEC in fully-hadronic W^+W^- events with those in semi-hadronic W^+W^- events ($q\bar{q}\ell\nu$), serves as a probe to study inter-W BEC. Together with colour reconnection [5, 6], inter-W BEC form a potential bias in the determination of the W mass at LEP.

In this Letter we search for evidence of inter-W BEC. We examine all BEC, intra-W as well as inter-W, and make a comparison with the BEC observed in hadronic Z decay with and without the contribution of $Z \rightarrow b\bar{b}$ decay. This Letter updates the analysis of Reference 7 using about 3.5 times more data.

Data and Monte Carlo

The data used in this analysis were collected by the L3 detector [8] at $\sqrt{s} = 189 - 209$ GeV and correspond to a total integrated luminosity of 629 pb^{-1} . These data are grouped in seven energy bins listed in Table 1.

Fully-hadronic and semi-hadronic W^+W^- events are selected with criteria similar to those described in Reference 9. An additional requirement for the fully-hadronic channel is a cut on the neural network output [9] to further separate the signal from the dominant $e^+e^- \rightarrow q\bar{q}(\gamma)$ background. In total, about 3,800 semi-hadronic and 5,100 fully-hadronic events are selected.

The event generator KORALW [10] with the BEC algorithm BE₃₂ [4] is used to simulate the signal process. The values of the BE₃₂ parameters are found by tuning the Monte Carlo (MC) to Z-decay data depleted in b-quark events. Both the BEC and the fragmentation parameters are tuned simultaneously ¹⁾. This set is used in this analysis. Systematic studies are made using an alternative set of parameter values ²⁾, obtained by tuning to Z-decay data of all flavours and used in Reference 7.

The background processes $e^+e^- \rightarrow q\bar{q}(\gamma)$, $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow Ze^+e^-$ are generated using PYTHIA [11]. For the $q\bar{q}(\gamma)$ channel KK2f [12] is also used. BEC are included in both programs. The generated events are passed through the L3 detector simulation program [13], reconstructed and subjected to the same selection criteria as the data.

The selection efficiencies of the W^+W^- channels $q\bar{q}e\nu$, $q\bar{q}\mu\nu$, $q\bar{q}\tau\nu$ and $q\bar{q}q\bar{q}$ are found to be 83%, 75%, 50% and 86%, respectively. The purities of these channels are around 95%, 95%, 85% and 80%, respectively, varying a few percent between the different energy bins. The selection efficiencies of fully-hadronic events changes by less than 0.5% when BEC (intra-W, or both intra-W and inter-W) are not simulated.

A sample of hadronic Z-decay events is selected from the data collected by the L3 detector at $\sqrt{s} = 91.2$ GeV, corresponding to a total integrated luminosity of 48.1 pb^{-1} . The event selection, which is similar to that presented in Reference 2, results in about one million hadronic Z-decay events with a purity greater than 99%. Since b quarks are greatly suppressed in W decays, and to compare with the W^+W^- data, a light-quark (u, d, s or c) enhanced Z-decay

¹⁾ The BEC parameter values PARJ(92)=0.90, giving the BEC strength, and PARJ(93)=0.43 GeV, the inverse effective source size, are obtained.

²⁾ The alternative set uses the BEC parameter values PARJ(92)=1.68 and PARJ(93)=0.38 GeV.

sample is selected using an anti-b tagging procedure [14] to reduce the $b\bar{b}$ fraction from 22% to 3%. This results in about 180,000 Z-decay events depleted in b quarks.

The charged pions used for the BEC study are detected as tracks in the central tracker, using selection criteria similar to those of Reference 7. About 82% of the tracks selected in MC samples are pions. This selection yields about one million pairs of like-sign particles in the fully-hadronic channel and about 200,000 pairs in the semi-hadronic channel.

Analysis Method

The Two-Particle Bose-Einstein Correlation Function

BEC are due to interference between identical bosons which are close to one another in phase space. These correlations can be studied in terms of the two-particle correlation function

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)} , \quad (1)$$

where $\rho_2(p_1, p_2)$ is the two-particle number density of particles with four-momenta p_1 and p_2 , and $\rho_0(p_1, p_2)$ the same density in the absence of BEC. As is customary in correlation studies, the normalization is $\iint \rho_2(p_1, p_2) dp_1 dp_2 = \langle n(n - \delta_{12}) \rangle$, where n is the number of particles and δ_{12} is 1 for identical particles and 0 otherwise. The largest BEC occur at small absolute four-momentum difference $Q \equiv \sqrt{-(p_1 - p_2)^2}$ and R_2 is parametrized in this one-dimensional distance measure by defining

$$\rho_2(Q) = \frac{1}{N_{\text{ev}}} \frac{dn_{\text{pairs}}}{dQ} , \quad (2)$$

where N_{ev} is the number of selected events and n_{pairs} the number of like-sign track pairs in the N_{ev} events, retaining the normalization of $\rho_2(p_1, p_2)$.

Apart from the quark flavour, hadronic W and Z decays are expected to be similar. Hence, we measure the correlation function for a light-quark Z-decay sample, $R_2^{\text{Z} \rightarrow \text{uds}}$, in order to compare it with those of the fully-hadronic W^+W^- events, R_2^{WW} , and the semi-hadronic W^+W^- events, R_2^W .

If there is no inter-W interference, we can write [15]

$$\rho_2^{\text{WW}}(p_1, p_2) = 2\rho_2^W(p_1, p_2) + 2\rho_1^W(p_1)\rho_1^W(p_2) , \quad (3)$$

where the assumption is made that the densities for the W^+ and W^- bosons are the same. In order to transform from p_1, p_2 to Q , we replace the product of the single-particle densities by a two-particle density, ρ_{mix} , yielding $\rho_2^{\text{WW}}(Q) = 2\rho_2^W(Q) + 2\rho_{\text{mix}}^{\text{WW}}(Q)$. We obtain $\rho_{\text{mix}}^{\text{WW}}(Q)$ by pairing particles originating from two different semi-hadronic events. By construction, particles in these pairs are uncorrelated. The event mixing procedure is explained in detail in References 7 and 16. This procedure also takes into account the momentum correlation between the two W's. The other two densities, ρ_2^{WW} and ρ_2^W , are measured directly in the fully-hadronic and the semi-hadronic events, respectively.

For comparison to the actual W^+W^- data, R_2^{WW} is estimated for the case that inter-W BEC are absent. In this case

$$R_{2, \text{no inter}}^{\text{WW}}(Q) = \frac{\rho_2^{\text{WW}}(Q)}{\rho_0^{\text{WW}}(Q)} = \frac{2\rho_2^W(Q) + 2\rho_{\text{mix}}^{\text{WW}}(Q)}{2\rho_0^W(Q) + 2\rho_{\text{mix}}^{\text{WW}}(Q)} = 1 + (1 - F(Q)) (R_2^W(Q) - 1) , \quad (4)$$

where

$$F(Q) \equiv \frac{2\rho_{\text{mix}}^{\text{WW}}(Q)}{\rho_{\text{mix}}^{\text{WW}}(Q)} \quad (5)$$

is the fraction of pairs, in which the two particles come from different W's, that would occur in fully-hadronic events if there were no BEC. In Equation (4) we assume that $\rho_{\text{mix}}^{\text{WW}}$ is the same for a sample with and without intra-W BEC (and no inter-W BEC). From a MC study, this assumption is found to be valid. In the extreme case of complete overlap of the two-particle densities, $\rho_0^W = \rho_{\text{mix}}^{\text{WW}} = \frac{1}{4}\rho_0^{\text{WW}}$, and then $F(Q) = 0.5$. In the other extreme, namely no overlap, $\rho_{\text{mix}}^{\text{WW}} = 0$ at $Q = 0$, so that $F(Q) = 0$ there, with the necessary compensation at large Q . Then BEC are equally strong in fully- and semi-hadronic events. Thus, depending on the degree of overlap, the strength of the BEC in fully-hadronic events is reduced by up to 50% compared to that of semi-hadronic events. We estimate $F(Q)$ using a MC without BEC.

Direct Search for Inter-W BEC

The hypothesis that the two W's decay independently can be directly tested using Equation (3). In particular, the following test statistics are defined as the difference and the ratio of the left- and right-hand side of Equation (3) in terms of Q

$$\Delta\rho(Q) = \rho_2^{\text{WW}}(Q) - 2\rho_2^W(Q) - 2\rho_{\text{mix}}^{\text{WW}}(Q) \quad (6)$$

and

$$D(Q) = \frac{\rho_2^{\text{WW}}(Q)}{2\rho_2^W(Q) + 2\rho_{\text{mix}}^{\text{WW}}(Q)} \quad . \quad (7)$$

This method gives access to inter-W correlations directly from the data, with no need of MC [15].

In the absence of inter-W correlations, $\Delta\rho = 0$ and $D = 1$. To study inter-W BEC, deviations from these values are examined at small values of Q for like-sign particles. The influence of other correlations or potential bias on these quantities is studied by analysing unlike-sign pairs and MC events.

The event mixing procedure could introduce artificial distortions or not fully account for some correlations other than BEC or some detector effects, causing a deviation of $\Delta\rho$ from zero or D from unity for data as well as for a MC without inter-W BEC. These possible effects are reduced by using the double ratio

$$D'(Q) = \frac{D(Q)_{\text{data}}}{D(Q)_{\text{MC, no inter}}} \quad , \quad (8)$$

where $D(Q)_{\text{MC, no inter}}$ is derived from a MC sample without inter-W BEC.

Results

Measurement of the Correlation Function

To obtain the correlation function R_2 , Equation (1), for the W^+W^- events, background is subtracted by replacing $\rho_2(Q)$, Equation (2), by

$$\rho_2(Q) = \frac{1}{\mathcal{P}N_{\text{ev}}} \left(\frac{dn}{dQ} - \frac{dn_{\text{bg}}}{dQ} \right) \quad , \quad (9)$$

where \mathcal{P} is the purity of the selection and n_{bg} is the number of pairs of tracks corresponding to $(1 - \mathcal{P})N_{\text{ev}}$ background events. This density is further corrected for detector resolution, acceptance, efficiency and for particle misidentification with a multiplicative factor derived from MC. Since no hadrons are identified, this factor is the two-pion density found from MC events at generator level divided by the two-particle density found using all particles after full detector simulation, reconstruction and selection. For this detector correction, the no inter-W scenario with the BE₃₂ algorithm is used.

The reference distribution ρ_0 is calculated from a KORALW MC sample without any BEC at generator level, using only pions. We use a MC distribution rather than unlike-sign particle pairs from data, as these are strongly affected by the presence of dynamical correlations, such as resonances.

Figure 1 shows the correlation function for semi-hadronic and fully-hadronic events. Results for different energy bins are compatible with each other, and therefore are combined.

The correlation function of the light-quark Z-decay data sample, $R_2^{\text{Z} \rightarrow \text{udcs}}$, is determined similarly to R_2^W and R_2^{WW} . Figure 1a compares $R_2^{\text{Z} \rightarrow \text{udcs}}$ with R_2^W and, as expected, good agreement is observed. If b-quark decays of the Z are not removed from the sample, a depletion of the correlation function is observed at low Q , and a clear discrepancy exists with the W data, as also shown in Figure 1a.

The BEC enhancement at low Q values is fitted from $Q = 0$ to 1.4 GeV with the first-order Edgeworth expansion of the Gaussian [17, 2, 18],

$$R_2(Q) = \gamma(1 + \delta Q) \left[1 + \lambda \exp(-R^2 Q^2) \left(1 + \frac{\kappa}{6} H_3(\sqrt{2} R Q) \right) \right] \quad , \quad (10)$$

using the full covariance matrix [16], which takes into account the effect of bin-to-bin correlations. The parameter γ is an overall normalization factor, the term $(1 + \delta Q)$ takes into account possible long-range momentum correlations, λ measures the strength of the BEC, R is related to the effective source size in space-time, κ measures the deviation from the Gaussian and $H_3(x) \equiv x^3 - 3x$ is the third Hermite polynomial. The fit results are given in Table 2 and shown in Figure 1.

BEC are observed ($\lambda > 0$) in both fully-hadronic and semi-hadronic events. The value of λ is higher for the semi-hadronic than for the fully-hadronic channel, which could be due to a partial suppression of inter-W BEC [15] combined with incomplete overlap of $\rho_2^{\text{W}^+}$ and $\rho_2^{\text{W}^-}$. Using a MC without BEC to correct the data for detector effects, reduces λ in both channels by about 30%, but the difference in the λ values remains.

Different sources of systematic uncertainties on the fit parameters are studied. Track and event selections are varied, the background fractions are varied by $\pm 20\%$, covering biases in the Q distributions of background events, and different MC's, using different sets of MC parameter values, are used to generate the background events. Moreover, the stability of the results is tested by changing the fully-hadronic event selection such that the background fraction is reduced by more than a factor two.

The influence of the choice of the MC used for the reference sample and for the detector correction is also taken into account. For the detector correction a MC with intra-W BEC is used, since it agrees better with the Q distribution of the raw data than a MC without BEC [7]. Both sets of parameter values in BE₃₂ are used and for the fully-hadronic channel both the no inter-W and inter-W scenarios are considered. For the inter-W scenario, the systematic uncertainty is estimated using the fraction of inter-W BEC consistent with the measurements of $\Delta\rho$, D and D' discussed below. Finally, a systematic uncertainty is estimated by varying

the fit range by ± 320 MeV. The contributions to the systematic uncertainty of λ are shown in Table 3.

The correlation function for fully-hadronic events expected for the case of no inter-W BEC, $R_{2, \text{no inter}}^{\text{WW}}$, is estimated from Equation (4). In this calculation, the value of $F(Q)$, from Equation (5) using KORALW without BEC, is around 0.2 at low Q and increases to 0.6 for $Q > 3$ GeV. Using these values of $F(Q)$ and $R_2^W = R_2^{Z \rightarrow \text{udcs}}$ in Equation (4), we obtain the no inter-W prediction shown in Figure 1b as the full histogram. Good agreement is observed between this distribution and the fully-hadronic W^+W^- data, thus indicating no or only weak inter-W BEC.

Measurement of $\Delta\rho$, D and D' , Direct Search for Inter-W BEC

Figure 2 shows the distribution of $\Delta\rho$, Equation (6), for like-sign, (\pm, \pm) , and for unlike-sign, $(+, -)$, particle pairs. Figure 3 shows the distributions of D and D' , Equations (7) and (8), for like-sign and unlike-sign pairs. For the double ratio D' we use the no inter-W scenario of KORALW as the reference sample. The distributions of $\Delta\rho$, D and D' are not corrected for detector effects, but background is estimated from MC and subtracted according to Equation (9), from ρ_2^W and ρ_2^{WW} . Also shown in Figures 2 and 3 are the predictions of KORALW after full detector simulation, reconstruction and selection. Both the inter-W and no inter-W scenarios are shown.

The inter-W scenario shows an enhancement at small values of Q in the $\Delta\rho$, D and D' distributions for like-sign pairs. We also observe a small enhancement for unlike-sign pairs due to the MC implementation of BEC, which shifts the vector momentum of particles, affecting both the like-sign and unlike-sign particle spectra. The no inter-W scenario describes the $\Delta\rho(\pm, \pm)$, $D(\pm, \pm)$ and $D'(\pm, \pm)$ distributions, while the inter-W scenario is disfavoured.

For quantitative comparisons, the integral

$$J(\pm, \pm) \equiv \int_0^{Q_{\text{max}}} \Delta\rho(Q) \, dQ \quad (11)$$

is computed. Also, the $D'(\pm, \pm)$ distribution is fitted from $Q = 0$ to 1.4 GeV, using the full covariance matrix, with the parametrization

$$D'(Q) = (1 + \delta Q)(1 + \Lambda \exp(-k^2 Q^2)) \quad , \quad (12)$$

where δ , Λ and k are the fit parameters. Both $J(\pm, \pm)$ and Λ measure the strength of inter-W BEC.

The systematic uncertainties on $J(\pm, \pm)$ and on Λ are listed in Tables 4 and 5, respectively. In addition to the track and event selections, the amount of background is also varied and different MC's, using both sets of MC parameter values, are used to generate the background events. Furthermore, contributions to the systematic uncertainty on Λ are obtained by varying the choice of MC for the reference sample in D' using PYTHIA and KORALW, both with no BEC at all and with only intra-W BEC. MC's without BEC are used to estimate the effect of residual intra-W BEC. The effect of various models for colour reconnection ³⁾ is included. Changes in the fit range (± 400 MeV), in the bin size (from 40 to 80 MeV) and in the parametrization of Equation (12) (removing $(1 + \delta Q)$ from the fit) also contribute to the systematic uncertainty on Λ .

³⁾The so-called SKI (with reconnection probability of about 30%), SKII and SKII' [6] models, as implemented in PYTHIA, are used.

In the mixing procedure, a semi-hadronic event is allowed to be combined with all possible other semi-hadronic events. To be sure that this does not introduce a bias, the analysis is repeated for a mixed sample where every semi-hadronic event is used at most once. The influence of the mixing procedure is also studied by not only combining oppositely charged W's, but also like-sign W's. The influence of an extra momentum [7], used in the event mixing, is also included as a systematic effect. The effect of these three changes in the mixing procedure is also given in Tables 4 and 5. Moreover, the analysis is repeated removing the cut on the neural network output for the mixed events. Furthermore, the effect of uncertainties in the energy calibration of the calorimeters is studied. Finally, the influence of incorrect assignment of tracks to τ or $q\bar{q}$ systems in the $q\bar{q}\tau\nu$ channel is investigated.

The value of $J(\pm, \pm)$ is computed using the full covariance matrix, taking $Q_{\max} = 0.68$ GeV, the value where the two MC scenarios have converged to less than one standard deviation. The results for each centre-of-mass energy, displayed in Figure 4a, are consistent with each other. Combining all $J(\pm, \pm)$ values results in

$$J(\pm, \pm) = 0.03 \pm 0.33 \pm 0.15 \quad ,$$

where the first uncertainty is statistical and the second systematic. Using KORALW with the inter-W scenario gives $J(\pm, \pm) = 1.38 \pm 0.10$, where the uncertainty is statistical only. In Figure 4a this value is shown as a vertical band. It disagrees with the value of the data by 3.6 standard deviations. For unlike-sign pairs we obtain $J(+, -) = 0.01 \pm 0.36 \pm 0.16$, consistent with zero.

The value of the fit parameter Λ , Equation (12), is shown in Figure 4b for each energy bin. Combining all Λ values results in

$$\Lambda = 0.008 \pm 0.018 \pm 0.012 \quad ,$$

where the first uncertainty is statistical and the second systematic. The value of k is found to be $0.4 \pm 0.4 \pm 0.3$ fm and the correlation coefficient between Λ and k is 0.45. A similar fit is performed for the KORALW MC sample with inter-W BEC, resulting in $\Lambda = 0.098 \pm 0.008$, where the uncertainty is statistical only. In Figure 4b this value is shown as a vertical band. It disagrees with the value of the data by 3.8 standard deviations. Using the alternative set of MC parameters results in $J(\pm, \pm) = 1.78 \pm 0.10$ and $\Lambda = 0.126 \pm 0.008$, where the uncertainties are statistical only.

To summarize, an excess at small values of Q in the distributions of $\Delta\rho(\pm, \pm)$, $D(\pm, \pm)$ and $D'(\pm, \pm)$ is expected from inter-W BEC, but none is seen. These distributions agree well with KORALW using BE₃₂ without inter-W BEC, but not when inter-W BEC are included. We thus find no evidence for BEC between identical pions originating from different W's.

Acknowledgements

Clarifying discussions with Š. Todorova-Nová are gratefully acknowledged.

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[§] Supported by the German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie
[†] Supported by the Hungarian OTKA fund under contract numbers T019181, F023259 and T037350.
[¶] Also supported by the Hungarian OTKA fund under contract number T026178.
[♭] Supported also by the Comisión Interministerial de Ciencia y Tecnología.
[‡] Also supported by CONICET and Universidad Nacional de La Plata, CC 67, 1900 La Plata, Argentina.
[△] Supported by the National Natural Science Foundation of China.

$\langle \sqrt{s} \rangle$ (GeV)	188.6	191.6	195.5	199.5	201.8	204.8	206.6
Integrated luminosity (pb $^{-1}$)	176.8	29.8	84.1	83.3	37.2	79.0	139.1

Table 1: Average centre-of-mass energies and corresponding integrated luminosities.

Parameter	Fully-hadronic	Semi-hadronic
γ	$0.92 \pm 0.02 \pm 0.02$	$0.91 \pm 0.02 \pm 0.02$
λ	$0.74 \pm 0.04 \pm 0.06$	$0.95 \pm 0.08 \pm 0.05$
R (fm)	$0.73 \pm 0.03 \pm 0.06$	$0.71 \pm 0.04 \pm 0.07$
δ (GeV $^{-1}$)	$0.00 \pm 0.02 \pm 0.05$	$0.03 \pm 0.02 \pm 0.05$
κ	$0.80 \pm 0.09 \pm 0.10$	$0.61 \pm 0.14 \pm 0.11$
χ^2/dof	27.2/30	35.4/30

Table 2: Values of the fit parameters γ , λ , R , δ and κ for the fully-hadronic and semi-hadronic events. The first uncertainty is statistical, the second systematic. The χ^2 and number of degrees of freedom (dof) are also given.

Source	Fully-hadronic	Semi-hadronic
Track selection	0.021	0.016
Event selection	0.013	0.014
Background contribution	0.018	0.021
Alternative MC as reference	0.013	0.013
Alternative MC for correction	0.022	0.024
Inter-W BE in MC correction	0.032	-
Fit range	0.020	0.018
Total	0.06	0.05

Table 3: Contributions to the systematic uncertainty on the λ parameter for the fully-hadronic and semi-hadronic events.

Source	Contribution
Track selection	0.084
Event selection	0.068
Background contribution	0.055
Mixing procedure	0.065
Neural network cut	0.038
Energy calibration	0.024
Track misassignment in $q\bar{q}\tau\nu$ channel	0.038
Total	0.15

Table 4: Contributions to the systematic uncertainty of $J(\pm, \pm)$.

Source	Contribution
Track selection	0.0029
Event selection	0.0049
Background contribution	0.0042
Alternative MC as a reference	0.0060
Colour reconnection	0.0026
Fit range	0.0018
Rebinning	0.0020
Fit parametrization	0.0017
Mixing procedure	0.0044
Neural network cut	0.0033
Energy calibration	0.0017
Track misassignment in $q\bar{q}\tau\nu$ channel	0.0022
Total	0.012

Table 5: Contributions to the systematic uncertainty of the Λ parameter.

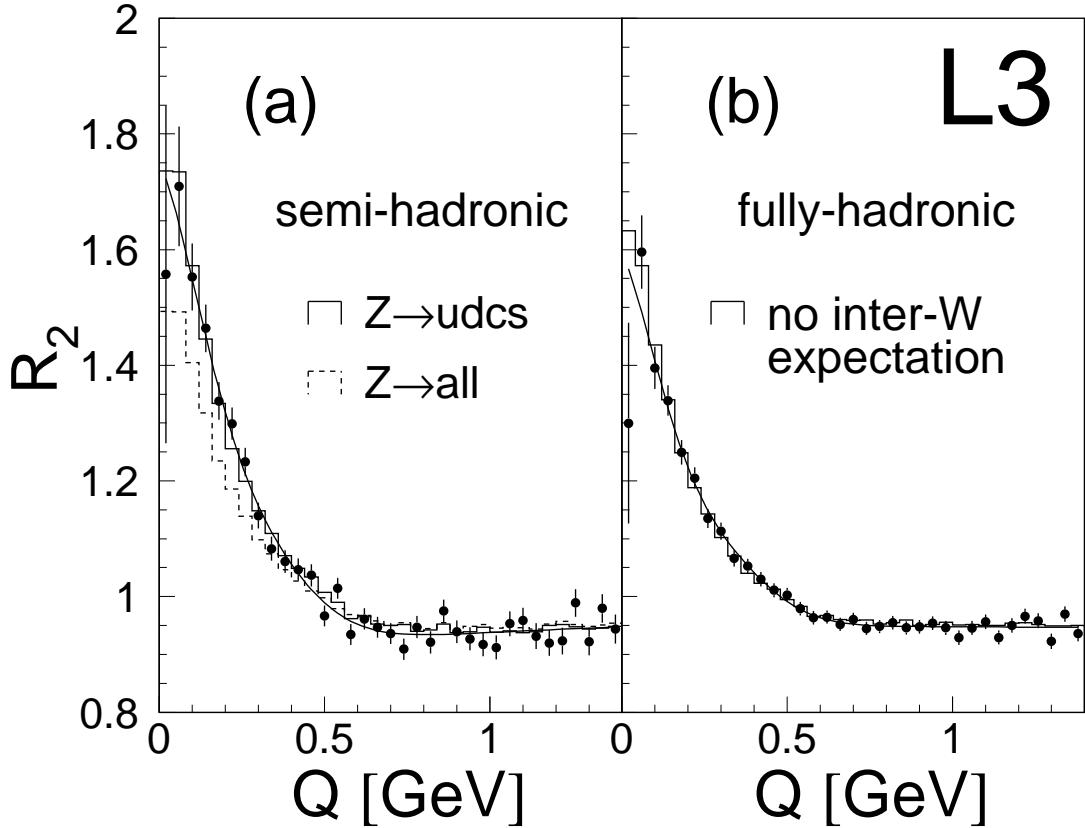


Figure 1: The correlation function R_2 at $\sqrt{s} = 189\text{--}209\text{ GeV}$, for (a) semi-hadronic events and (b) fully-hadronic events. The solid curves are the results from the fits of Equation (10) to the W^+W^- data. In (a) the full histogram is for the light-quark Z -decay data sample and the dashed histogram is for the data sample containing all hadronic Z decays. In (b) the full histogram gives the expectation when inter- W BEC are absent, Equation (4), using $R_2^W = R_2^{Z \rightarrow \text{udcs}}$.

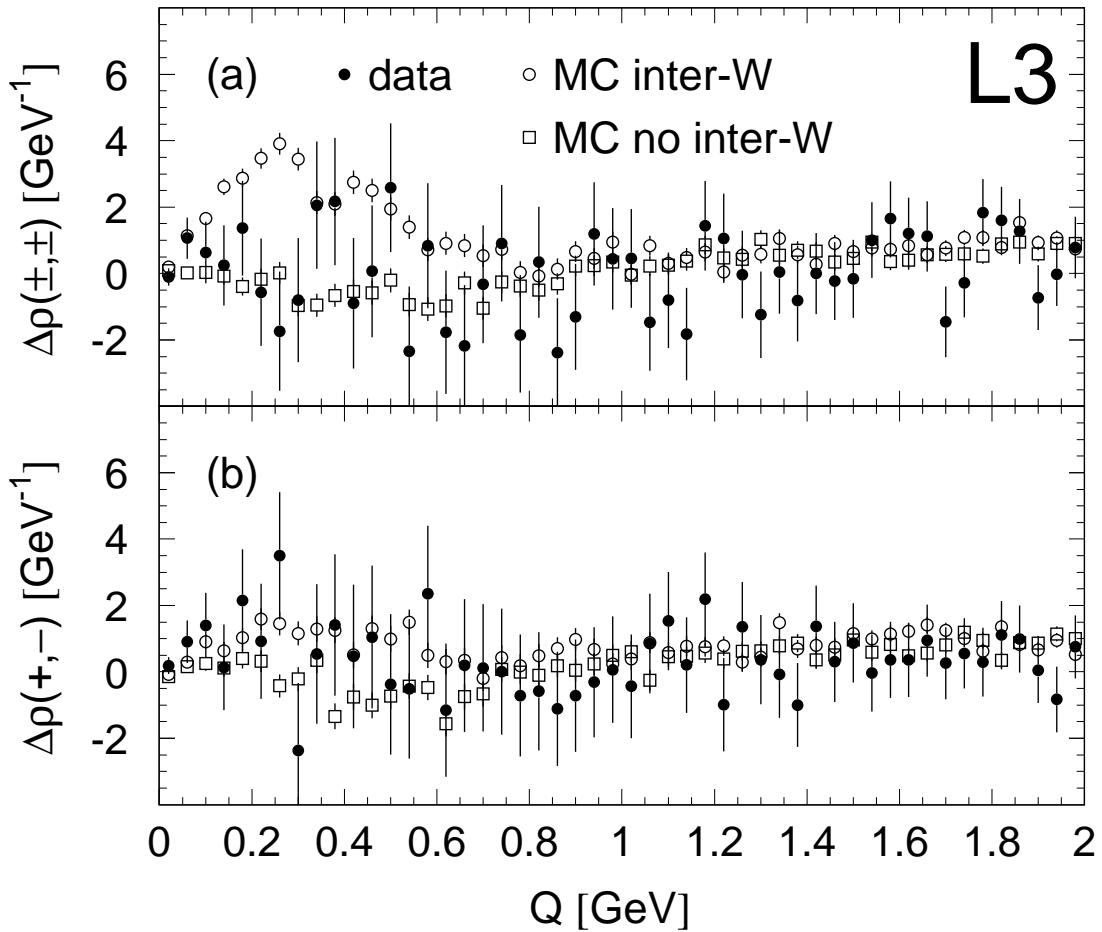


Figure 2: Distributions for uncorrected data at $\sqrt{s} = 189\text{--}209\text{ GeV}$ of (a) $\Delta\rho(\pm,\pm)$ and (b) $\Delta\rho(+,-)$. Also shown are the MC predictions of KORALW with and without inter-W BEC.

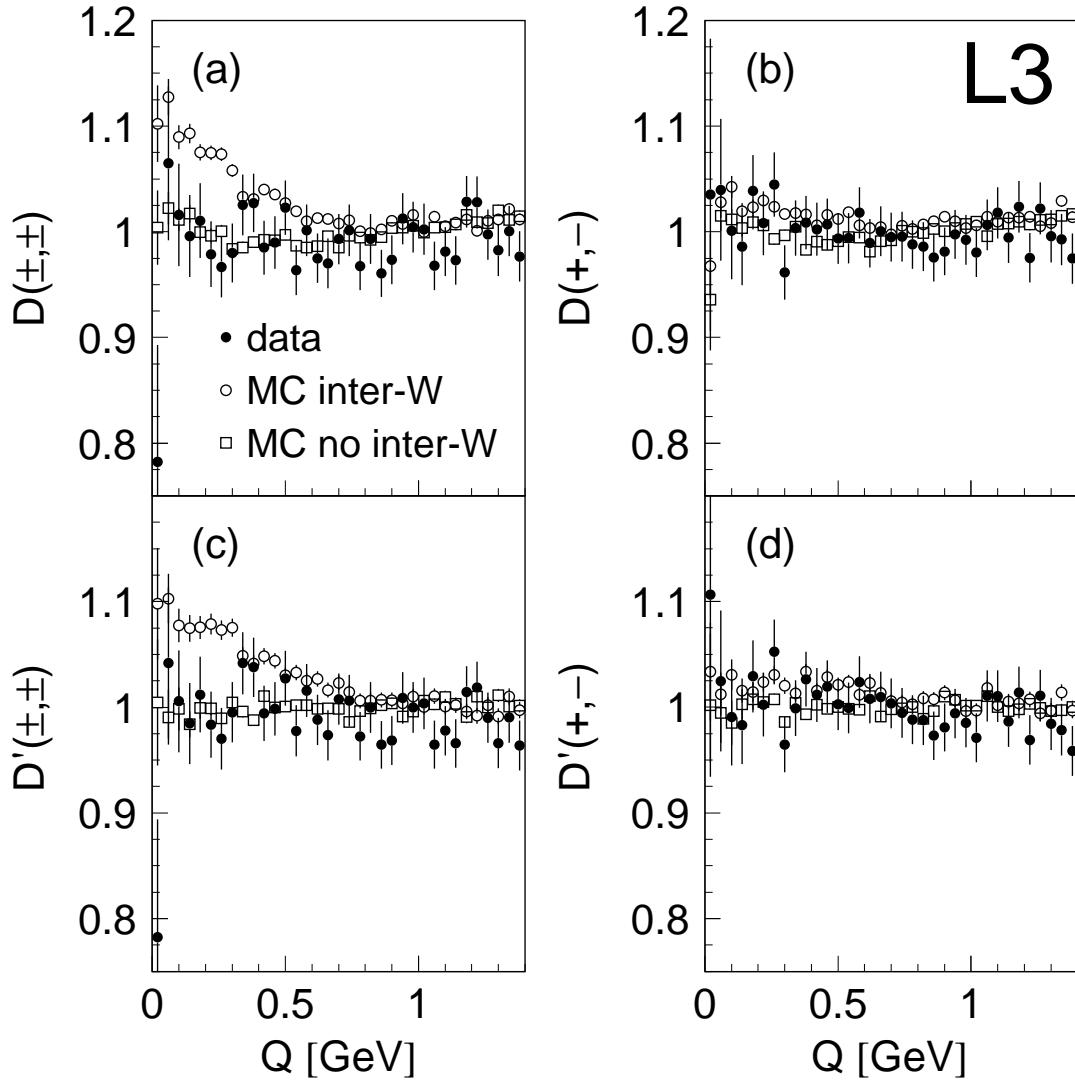


Figure 3: Distributions for uncorrected data at $\sqrt{s} = 189\text{--}209$ GeV of (a) $D(\pm, \pm)$, (b) $D(\pm, -)$, (c) $D'(\pm, \pm)$ and (d) $D'(\pm, -)$. Also shown are the MC predictions of KORALW with and without inter-W BEC.

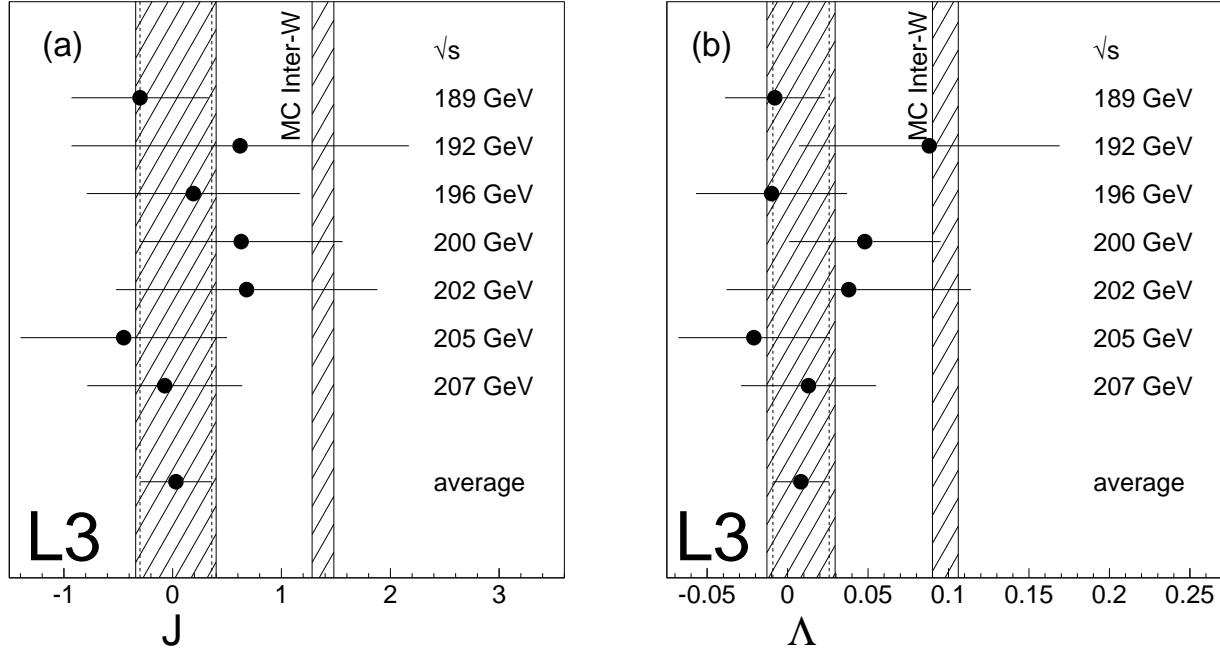


Figure 4: Values of (a) the integral $J(\pm, \pm)$ and (b) the Λ parameter, at different centre-of-mass energies and their average. The uncertainties are statistical only. The wide bands show the average value of the data including the systematic uncertainties. Also shown are the MC predictions of KORALW with inter-W BEC.